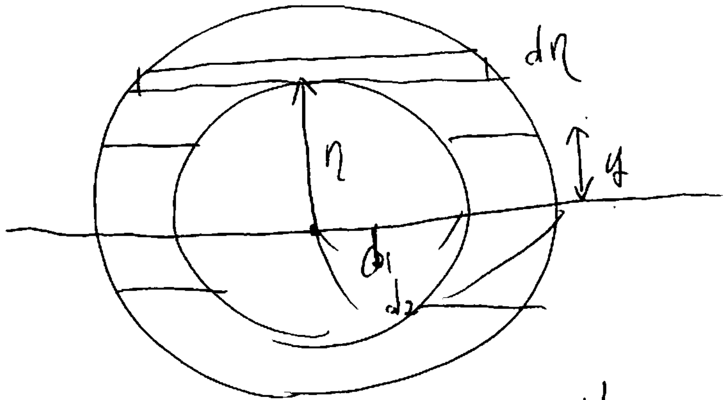

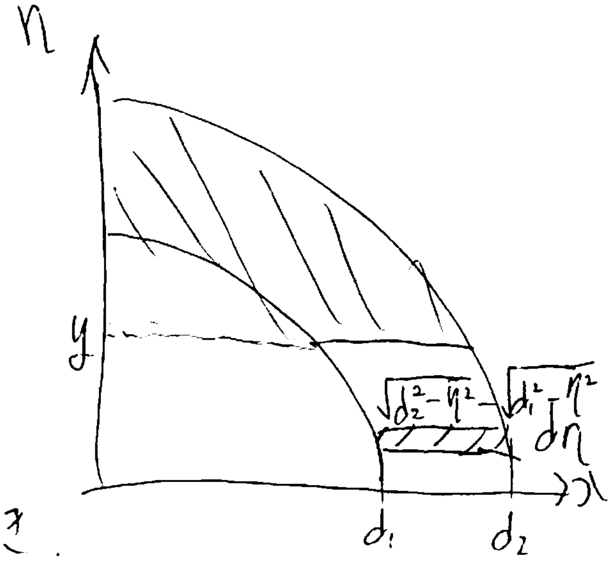

四分円で考える。 $y < d_1$ のとき

$$M = \int_A \sigma \eta \, dA$$

$$= 4\int_0^y \left(\sqrt{d_2^2-\eta^2} - \sqrt{d_1^2-\eta^2}\right) \cdot \sigma \eta \cdot d\eta + 4\int_y^{d_1} \left(\sqrt{d_2^2-\eta^2} - \sqrt{d_1^2-\eta^2}\right) \sigma_e \eta \, d\eta$$

$$+ 4\int_{d_1}^{d_2} \sqrt{d_2^2-\eta^2} \cdot \sigma_e \eta \, d\eta$$

$$= 4\int_0^y \left(\sqrt{d_2^2-\eta^2} - \sqrt{d_1^2-\eta^2}\right) \sigma_e \frac{\eta}{y} \cdot \eta \, d\eta + 4\int_y^{d_1} \left(\sqrt{d_2^2-\eta^2} - \sqrt{d_1^2-\eta^2}\right) \sigma_e \cdot \eta \, d\eta$$

$$+ 4\int_{d_1}^{d_2} \sqrt{d_2^2-\eta^2} \, \sigma_e \eta \cdot d\eta$$

弾性部分

(i) $$\int_0^y \left(\sqrt{d_2^2-\eta^2} - \sqrt{d_1^2-\eta^2}\right) \sigma_e \cdot \frac{\eta}{y} \cdot \eta \, d\eta = \frac{\sigma_e}{y} \left\{ \int_0^y \sqrt{d_2^2-\eta_1^2} \cdot \eta_1^2 \, d\eta_1 - \int_0^y \sqrt{d_1^2-\eta_2^2} \cdot \eta_2^2 \, d\eta_2 \right.$$

$\eta_1 = d_2 \sin\theta$ とおくと $d\eta_1 = d_2 \cos\theta \, d\theta$

η_1	$0 \to y$
θ	$0 \to \sin^{-1}\frac{y}{d_2}$

$\eta_2 = d_1 \sin\theta$ $\quad d\eta_2 = d_1 \cos\theta \, d\theta$

η_2	$0 \to y$
θ	$0 \to \sin^{-1}\frac{y}{d_1}$

$$= \frac{\sigma_e}{y} \left\{ \int_0^y \sqrt{d_2^2 - d_2^2 \sin^2\theta} \cdot d_2^2 \sin^2\theta \, d_2 \cos\theta \, d\theta - \int_0^y \sqrt{d_1^2 - d_1^2 \sin^2\theta} \cdot d_1^2 \sin^2\theta \, d_1 \cos\theta \, d\theta \right\}$$

$$= \frac{\sigma_e}{y} \left\{ \int_0^y d_2^4 \sin^2\theta \cos^2\theta \, d\theta - \int_0^y d_1^4 \sin^2\theta \cos^2\theta \, d\theta \right\}$$

$$= \frac{\sigma_e}{y}\left\{\int_0^{\sin^{-1}\frac{y}{d_2}} d_2^4 \cdot \frac{1}{4}\sin^2 2\theta\, d\theta - \int_0^{\sin^{-1}\frac{y}{d_1}} d_1^4 \cdot \frac{1}{4}\sin^2\theta\, d\theta\right\}$$

$$= \frac{\sigma_e}{y}\left\{\int \frac{d_2^4}{8}(1-\cos 4\theta)\, d\theta - \int \frac{d_1^4}{8}(1-\cos 4\theta)\, d\theta\right\}$$

$$= \frac{\sigma_e}{8y}\left\{d_2^4\left[\theta - \frac{1}{4}\sin 4\theta\right]_0^{\sin^{-1}\frac{y}{d_2}} - d_1^4\left[\theta - \frac{1}{4}\sin 4\theta\right]_0^{\sin^{-1}\frac{y}{d_1}}\right\}$$

$$= \frac{\sigma_e}{8y}\left[d_2^4\left\{\sin^{-1}\frac{y}{d_2} - \frac{1}{4}\sin\left(4\sin^{-1}\frac{y}{d_2}\right)\right\} - \left\{d_1^4 \sin^{-1}\frac{y}{d_1} - \frac{1}{4}\sin\left(4\sin^{-1}\frac{y}{d_1}\right)\right\}\right]$$

(ii) 塑性部分

$$\int_y^{d_1}\left(\sqrt{d_2^2-\eta^2} - \sqrt{d_1^2-\eta^2}\right)\sigma_e \cdot \eta\, d\eta + \int_{d_1}^{d_2}\sqrt{d_2^2-\eta^2}\,\sigma_e \cdot \eta\, d\eta$$

$$= \int_y^{d_2}\sqrt{d_2^2-\eta_1^2}\,\sigma_e \cdot \eta_1\, d\eta_1 - \int_y^{d_1}\sqrt{d_1^2-\eta_2^2}\,\sigma_e\, \eta_2\, d\eta_2$$

$\eta_1 = d_2\sin\theta$ とおくと $d\eta_1 = d_2\cos\theta\, d\theta$

$\eta_2 = d_1\sin\theta$ $\quad d\eta_2 = d_1\cos\theta\, d\theta$

η_1	$y \to d_2$
θ	$\sin^{-1}\frac{y}{d_2} \to \frac{\pi}{2}$

η_2	$y \to d_1$
θ	$\sin^{-1}\frac{y}{d_1} \to \frac{\pi}{2}$

$$= \int_{\sin^{-1}\frac{y}{d_2}}^{\frac{\pi}{2}} \sigma_e d_2^3 \cos^2\theta \sin\theta\, d\theta - \int_{\sin^{-1}\frac{y}{d_1}}^{\frac{\pi}{2}} \sigma_e d_1^3 \cos^2\theta \sin\theta\, d\theta$$

$$= \sigma_e d_2^3\left[-\frac{1}{3}\cos^3\theta\right]_{\sin^{-1}0}^{\frac{\pi}{2}} - \sigma_e d_1^3\left[-\frac{1}{3}\cos^3\theta\right]_{\sin^{-1}0}^{\frac{\pi}{2}}$$

$$= \sigma_e d_2^3\, \frac{1}{3}\left(1-\frac{y^2}{d_2^2}\right)^{\frac{3}{2}} - \sigma_e d_1^3 \frac{1}{3}\left(1-\frac{y^2}{d_1^2}\right)^{\frac{3}{2}}$$

$$= \sigma_e \frac{1}{3}(d_2^2-y^2)^{\frac{3}{2}} - \sigma_e \frac{1}{3}(d_1^2-y^2)^{\frac{3}{2}}$$

$$= \frac{1}{3}\sigma_e\left\{(d_2^2-y^2)^{\frac{3}{2}} - (d_1^2-y^2)^{\frac{3}{2}}\right.$$

(i)(ii) より

$$M = 4\frac{\sigma_e}{8y}\left[d_2^4\left\{\sin^{-1}\frac{y}{d_2} - \frac{1}{4}\sin\left(4\sin^{-1}\frac{y}{d_2}\right)\right\} - d_1^4\left\{\sin^{-1}\frac{y}{d_1} - \sin\left(4\sin^{-1}\frac{y}{d_1}\right)\right\}\right]$$

$$+ 4\cdot\frac{1}{3}\sigma_e\left\{(d_2^2 - y^2)^{\frac{3}{2}} - (d_1^2 - y^2)^{\frac{3}{2}}\right\}$$

ここで $y \to 0$ とすると

$\sin^{-1}\frac{y}{d_2} = 0$ $\sin^{-1}\frac{y}{d_1} = 0$ より

$$M_0 = \frac{4}{3}\sigma_e(d_2^3 - d_1^3) = \frac{1}{6}\sigma_e(D_2^3 - D_1^3)$$ //

問3(d) では

d_1, d_2 は直径のため 答えが $\frac{1}{6}\sigma_e(d_2^3 - d_1^3)$

となっている